or

$$\gamma-\beta^2=\beta\sqrt{\beta^2-1}\ \cos(\pi/n).$$

Substituting for γ, we obtain

$$\alpha=\frac{\mu}{\sqrt{2\mu-1}}$$

where

$$\mu=\beta\left[\beta+\sqrt{\beta^2-1}\ \cos(\pi/n)\right].$$

Once the value of α is known, we can compute $x_{\min}^2$ from (3). The modified transfer function is then given by

$$|s_{12}|^2=\left[\frac{1}{1+\epsilon^2F_n^2(x)}\right]_{x^2=(1-x_{\min}^2)y^2+x_{\min}^2}.$$

We now make the substitution

$$y^2=\frac{\beta^2\Omega^2}{1+\Omega^2}$$

where

$$\Omega=\tan\left(\frac{\pi}{2}\cdot\frac{\omega}{\omega_0}\right)$$

and synthesize the network in a straightforward manner [6]. (As a matter of fact, the synthesis is somewhat easier in the even-order case because we can derive explicit formulas for the poles and zeros of the reflection factor $s_{11}(\lambda)$.)

III. Some Additional Comments

In [2] Gupta and Wenzel have shown that in the wide-band odd-order case, the optimum design method yields a network with a substantially improved performance in the stopband when compared with the redundant design method of [1]. The same conclusion holds in the case of wide-band even-order filters also. As an example, consider the case of a 4-element 100-percent bandwidth filter designed by the present method (maximum VSWR in the passband is 1.1). The stopband attenuation response of the filter is shown in Fig. 2. For comparison the attenuation responses of a 3-element optimum filter and a 4-element redundant unit-element filter are also shown. Clearly, the 4-element optimum filter gives an improved performance in the stopband.

References

[1] B. M. Schiffman and G. L. Matthaei, "Exact design of band-stop microwave filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 6–15, Jan. 1964.
[2] O. P. Gupta and R. J. Wenzel, "Design tables for a class of optimum microwave bandstop filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 402–404, July 1970.
[3] H. J. Carlin and W. Kohler, "Direct synthesis of band-pass transmission line structures," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 283–297, May 1965.
[4] R. Saal and E. Ulbrich, "On the design of filters by synthesis," *IRE Trans. Circuit Theory*, vol. CT-5, pp. 284–327, Dec. 1958.
[5] L. F. Lind, "Synthesis of equally terminated lowpass lumped and distributed filters of even order," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 43–45, Jan. 1969.
[6] H. J. Carlin, "Distributed circuit design with transmission line elements," *Proc. IEEE*, vol. 59, pp. 1059–1081, July 1971.

Interdigital Microstrip Circuit Parameters Using Empirical Formulas and Simplified Model

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***Abstract*—Empirical formulas are given for single- and multiple-coupled microstrips, directly giving propagation mode admittances and phase velocities to be used in simplified expressions for the admittance parameters for the 2*N*-port network in the form of *N*-coupled strips, thus forming the basis for both analysis and synthesis of interdigital microstrip circuits. Calculated and measured results are presented for two interdigital bandpass filters synthesized as Chebyshev filters.**

Manuscript received May 23, 1978; revised October 10, 1978.
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I. Introduction

In the analysis and synthesis of coupled microstrip circuits, a major problem is how to obtain the primary parameters, that is, propagation mode admittances and phase velocities. For a single strip and for two identical coupled strips, formulas exist which give the propagation mode parameters as functions of the relative dielectric constant ϵ_r and the physical dimensions [2]–[4]. For multiple-coupled strips various numerical methods,

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e.g., [6] can be used to calculate the capacitance matrices for the system with and without dielectric. From these the desired parameters can be found [7], [8]. However, in an iterative synthesis procedure, this method is much too time consuming.

The aim of this work was to develop formulas for multiple coupled strips, which could be used in an iterative synthesis procedure. If such a procedure is to be convergent, the formulas must give results that vary in a way predictable by physical reasoning, when some dimension is changed.

In the following, empirical formulas are presented for a single strip, two identical coupled strips, and multiple-coupled strips, directly giving propagation mode parameters to be used in simplified expressions for the admittance parameters for the $2N$-port network N-coupled strips form. Lossfree and approximately TEM-mode propagation is assumed.

The empirical formulas are valid for open microstrip circuits, but it should be possible to develop similar formulas for covered circuits also.

II. Single Microstrip

The electrical field around the strip is divided into seven parts, each corresponding to a capacitor as shown in Fig. 1. The strip capacitance C, the characteristic admittance Y, and the phase velocity v are then found as

$$C = C_0 + 2\cdot C_1 + 2\cdot C_2 + 2\cdot C_3, \qquad \text{F/m} \tag{1}$$

$$Y = v_0 \cdot \left(\frac{C_0 + 2\cdot C_1}{\sqrt{\epsilon_r}} + \frac{2\cdot C_2}{\sqrt{\epsilon_{r2}}} + \frac{2\cdot C_3}{\sqrt{\epsilon_{r3}}} \right), \qquad \text{mho} \tag{2}$$

$$v = Y/C, \qquad \text{m/s} \tag{3}$$

$$\epsilon_{r\,\mathrm{eff}} = (v_0/v)^2 \tag{4}$$

where ϵ_r = rel.diel.const. and $v_0 = 3\times10^8$ m/s.

C_0 represents the homogeneous electrical-field capacitance, C_1 and C_2 are fringing capacitances with relative dielectric constants ϵ_r and ϵ_{r2}, respectively, and C_3 is the extra fringing capacitance caused by the nonzero strip thickness.

The strip capacitance C (1) is found as the parallel connection of the seven capacitors, and the characteristic admittance Y (2) is found as the sum (parallel connection) of the admittances corresponding to the capacitors.

The capacitances C_0, C_1, C_2, and C_3 should vary in a way, which as closely as possible corresponds to the way the corresponding parts of the electrical field vary with physical parameters (dimensions and dielectric constant).

Because the model mirrors the physical reality to a high degree, it is possible to use the single-strip formulas as the basis for coupled-strip formulas.

The development of the empirical formulas given below started with the homogeneous thin strip case ($\epsilon_r = 1$, $t = T/H = 0$). Here only one function $f_1(w = W/H)$ (9) is needed to calculate the characteristic admittance Y. Here $f_1(w)$ will represent the sum of the fringing capacitances. It

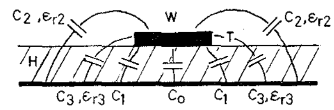


Fig. 1. The electrical field around a single microstrip.

was found (like the other empirical functions) by plotting desired function points found from the results obtained by others such as Wheeler [1], and then fitting some function to these points.

To divide the sum of the fringing capacitances into C_1 and C_2 was a problem which was solved by mapping (approximately) the electrical field around the strip for a few values of $w = W/H$. In the homogeneous case it was found that $C_1 \simeq C_2/2$ over a wide range of values for the stripwidth.

The function $f_1(w)$ can also be viewed as the additional relative stripwidth necessary to give the correct characteristic admittance, assuming homogeneous electrical field and $\epsilon_r = 1$. For $\epsilon_r \neq 1$ this additional relative stripwidth is different from $f_1(w)$ by a factor $f_2(w, \epsilon_r)$ (10). After finding this function in the same way as $f_1(w)$ was found, the problem was again to find C_1 and C_2 separately. Here it was found that the expression given in (6) for C_1 gave monotonic functions in ϵ_r, and $w = W/H$ for C_2, and its apparent relative dielectric constant ϵ_{r2}. This indicates that the choice in (6) cannot be far wrong. The results obtained for coupled strips also indicate that this choice is a good one.

The two functions f_1 and f_2 are all that is needed to calculate the single strip parameters when $t = T/H = 0$. However, the accuracy of the calculated phase velocity was improved by introducing a third function $f_3(w, \epsilon_r)$ to give the apparent relative dielectric constant ϵ_{r2} of C_2, $(1 < \epsilon_{r2} < \epsilon_r)$.

The accuracy of the single-strip formulas given below is on the same order as the accuracy of the formulas developed by Wheeler [1]. For values of w outside the interval $0.1 < w < 10$ and for $\epsilon_r > 16$ the accuracy decreases. The formulas, however, give results consistent with physical reasoning also outside these limits for ϵ_r and w. This is important because the single-strip formulas form the basis for coupled-strip formulas used in iterative synthesis computer programs for interdigital bandpass filters and for directional couplers. These synthesis procedures will not be convergent if the formulas do not give results consistent with physical reasoning also outside the intervals of practical values for the physical dimensions.

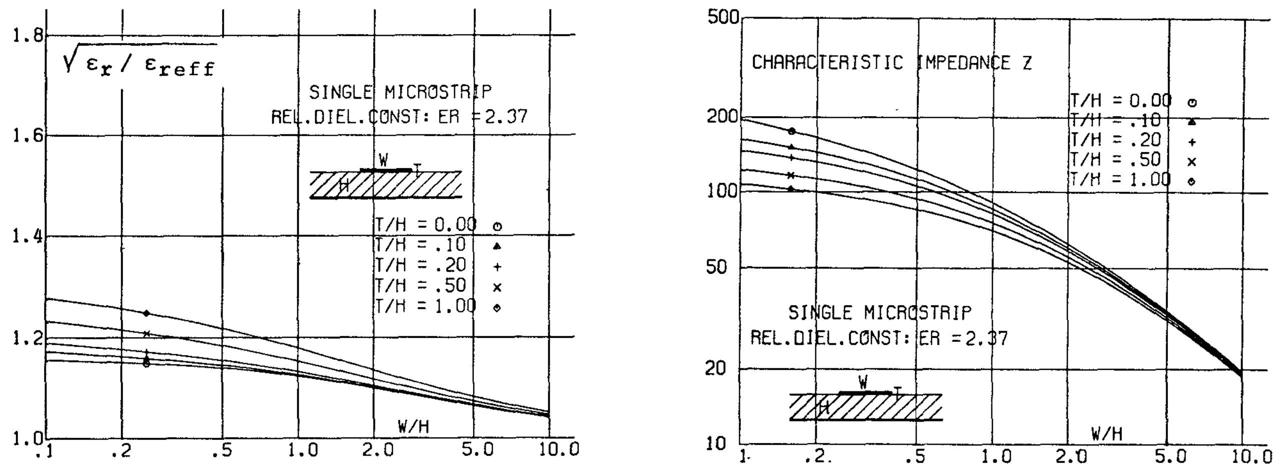
Thin Strip

In this case $C_3 = 0$. C_0, C_1, C_2, and ϵ_{r2}, which must be known in order to find C, Y, and v (1)–(3), can be found as

$$C_0 = C_0(w, \epsilon_r) = \epsilon_r \cdot \epsilon_0 \cdot w, \quad \epsilon_0 = 10^{-9}/(36\cdot\pi) \tag{5}$$

$$C_1 = C_1(w, \epsilon_r) = \epsilon_r \cdot \epsilon_0 \cdot f_1(w)/6 \tag{6}$$

$$\epsilon_{r2} = \epsilon_{r2}(w, \epsilon_r) = f_3(w, \epsilon_r) \tag{7}$$



(a) (b)

Fig. 2. (a) Relative phase velocity $=\sqrt{\epsilon_r/\epsilon_{reff}}$ for a single microstrip. (b) Characteristic impedance for a single microstrip.

$$C_2 = C_2(w,\epsilon_r) = \epsilon_0 \cdot \epsilon_{r2} \cdot (f_2(w,\epsilon_r) - 1/3) \cdot f_1(w) \cdot \sqrt{\epsilon_r/\epsilon_{r2}}\;/2 \tag{8}$$

where

$$f_1(w) = 1.68 + 0.23 \cdot \ln\left(1 + (w/0.51)^{1.924}\right) - 0.115 \cdot \ln\left(1 + (0.51/w)^{1.924}\right) \tag{9}$$

$$f_2(w,\epsilon_r) = 1 - \left(1 - \epsilon_r^{-0.69}\right) \cdot (0.3 + \ln(10 \cdot w)/50) \tag{10}$$

$$f_3(w,\epsilon_r) = \epsilon_r^x, \quad x = (0.35 + 0.32 \cdot \exp(-4.2/\epsilon_r))/(1+w). \tag{11}$$

Thick Strip

For a single strip with nonzero thickness ($t = T/H \neq 0$), C_2 and ϵ_{r2} are modified, and the capacitor C_3 is introduced. This capacitance represents the electrical-field lines starting on the side of the strip and must, therefore, have an apparent relative dielectric constant ϵ_{r3} greater than 1 but smaller than ϵ_r. The two formulas (14) and (15) giving C_3 and ϵ_{r3} are quite simple, but together with the modifications of C_2 and ϵ_{r2} (12), (13), they give results which are on the same order of accuracy as the results presented by Wheeler on thick microstrip [1]:

$$C_2 = C_2(w + 2 \cdot t, \epsilon_r) \tag{12}$$

$$\epsilon_{r2} = \epsilon_{r2}(w + 2 \cdot t, \epsilon_r) \tag{13}$$

$$C_3 = C_3(t,\epsilon_r) = 0.75 \cdot t/(t/\epsilon_0 + 1/\epsilon_0/\epsilon_r) \tag{14}$$

$$\epsilon_{r3} = (t+1)/(t + 1/\epsilon_r). \tag{15}$$

In Fig. 2 the characteristic impedance $(1/Y)$ and the relative phase velocity $v/(v_0/\sqrt{\epsilon_r}) = \sqrt{\epsilon_r/\epsilon_{reff}}$ are presented as functions of $w = W/H$ and $t = T/H$, for $\epsilon_r = 2.37$. From Fig. 2(a) it can be seen that the phase velocity becomes higher with the thickness of the strip, just as it should.

III. Two Identical Coupled Strips

In this case two propagation modes exist, the even mode and odd mode [2]–[5]. The two modes are shown in Figs. 3 and 4, where the electrical field around each strip is divided into five parts, each corresponding to a capacitor.

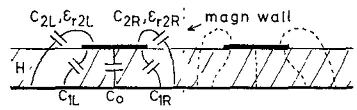


Fig. 3. Even mode. $Y = Y_E$, $C = C_E$, $v = v_E$.

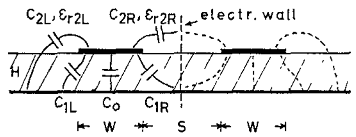


Fig. 4. Odd mode. $Y = Y_O$, $C = C_O$, $v = v_O$.

The strip thickness is assumed small here. It should be possible to take the strip thickness into account for coupled strips also, but so far it has only been done with passable accuracy in the odd-mode case, which by the way is the most important. The results for coupled thick strips, however, will not be given here.

The strip capacitance C, the characteristic admittance Y, and the phase velocity v are found as

$$C = C_0 + C_{1L} + C_{2L} + C_{1R} + C_{2R}, \quad \text{F/m} \tag{16}$$

$$Y = \left(\frac{C_0 + C_{1L} + C_{1R}}{\sqrt{\epsilon_r}} + \frac{C_{2L}}{\sqrt{\epsilon_{r2L}}} + \frac{C_{2R}}{\sqrt{\epsilon_{r2R}}} \right) \cdot v_0, \quad \text{mho} \tag{17}$$

$$v = Y/C, \quad \text{m/s}. \tag{18}$$

C_{1L}, C_{2L}, C_{1R}, C_{2R}, ϵ_{r2L}, and ϵ_{r2R} are naturally different for the two modes. They are found by modifying the formulas for a single strip as shown below.

Even Mode

$$C_0 = C_0(w,\epsilon_r) \tag{19}$$

$$C_{1L} = C_1(w',\epsilon_r) \tag{20}$$

$$\epsilon_{r2L} = \epsilon_{r2}(w',\epsilon_r) \tag{21}$$

$$C_{2L} = C_2(w',\epsilon_r) \tag{22}$$

$$C_{1R}=C_1(w,\epsilon_r)\cdot f_4(w,s) \quad (23)$$

$$C_{2R}=C_2(w,\epsilon_r)\cdot f_5(w,s) \quad (24)$$

$$\epsilon_{r2R}=\epsilon_{r2}(w,\epsilon_r) \quad (25)$$

where

$$w'=w\cdot(1+1/(1+s/w)) \quad (26)$$

$$s=S/H \quad (27)$$

$$f_4(w,s)=\left(1+0.56/(0.244\cdot\exp(2.5\cdot s)+w^{-2.59})^{0.54}\right)\cdot f_a \quad (28)$$

$$f_5(w,s)=(f_b+(1-f_b)\cdot\exp(-w^{1.5}/1.8/(1-f_b))\cdot f_a \quad (29)$$

and

$$f_a=f_a(w,s)=1-\exp\left(-(s/(1.26\cdot w^{0.24}))^x\right)$$

$$x=(1.13+0.1783\cdot\ln(w)+0.73\cdot s)/(1+s) \quad (30)$$

$$f_b=f_b(s)=(1+0.5715/s^{1.38})^{-0.76}. \quad (31)$$

Odd Mode

$$C_0=C_0(w,\epsilon_r) \quad (32)$$

$$C_{1L}=C_1(w,\epsilon_r) \quad (33)$$

$$C_{2L}=C_2(w,\epsilon_r) \quad (34)$$

$$C_{1R}=C_1(w,\epsilon_r)\cdot f_6(w,s) \quad (35)$$

$$\epsilon_{r2L}=\epsilon_{r2}(w,\epsilon_r) \quad (36)$$

$$C_{2R}=C_2(w,\epsilon_r)\cdot f_7(w,s) \quad (37)$$

$$\epsilon_{r2R}=\epsilon_{r2}(w,\epsilon_r)\cdot f_8(s,\epsilon_r) \quad (38)$$

where

$$f_6(w,s)=1+0.87\cdot\ln\left(1+(f_c/s)^{1.56}\right) \quad (39)$$

$$f_7(w,s)=1+0.87\cdot\ln\left(1+(f_d/s)^{1.56}\right) \quad (40)$$

$$f_8(s,\epsilon_r)=1-(\epsilon_r^{0.0414}-1)\cdot\exp(-2.5\cdot s) \quad (41)$$

and

$$f_c=-0.043\cdot\ln\left(1+(f_e/w)^{3.84}\right)+0.706+0.043\cdot\ln(s) \quad (42)$$

$$f_e=\exp(-0.388+0.261\cdot\ln(s)) \quad (43)$$

$$f_d=-0.043\cdot\ln\left(1+(f_f/w)^{3.84}\right)+1.01+0.043\cdot\ln(s) \quad (44)$$

$$f_f=\exp(-0.727+0.261\cdot\ln(s)). \quad (45)$$

In the development of the functions f_4–f_8, it was an important consideration that they should become correct for s going towards either 0 or ∞ and at the same time give the correct values for impedances and phase velocities in the normal range of values for w and s. That this goal was achieved to a high degree can be seen from Figs. 5(a), (b), and (6) in which even- and odd-mode impedances, phase velocities, and inductances are shown as functions of w and s. In Fig. 5(a) and (b) $\epsilon_r=10$, while in

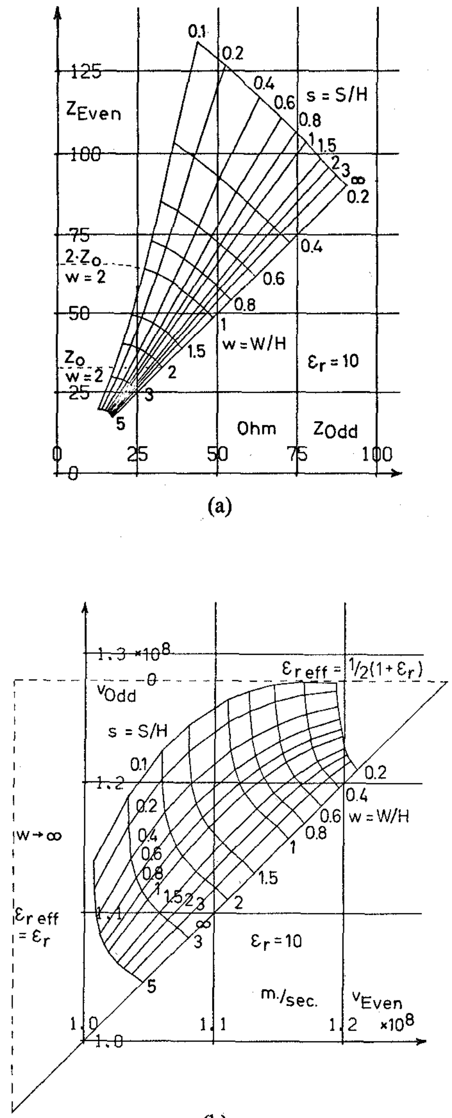


Fig. 5. (a) Even- and odd-mode impedances for two identical coupled strips. (b) Even- and odd-mode phase velocities for two identical coupled strips.

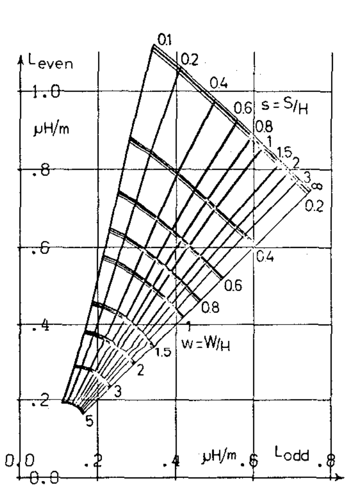


Fig. 6. Even- and odd-mode inductances for two identical coupled strips. 5 sets of curves for $\epsilon_r=1$, 2, 5, 10, and 20 have been superimposed on each other.

Fig. 6 the five sets of curves for $\epsilon_r = 1$, 2, 5, 10, and 20 have been superimposed on each other.

The way the results are presented readily shows that the parameters vary in a way predictable by physical reasoning, and that the impedances and inductances attain the correct values for s going towards either 0 or ∞. This is also true for the even-mode phase velocity, while the odd-mode phase velocity may become too high for s going towards 0; the largest value for the phase velocities being $v_0/\sqrt{\frac{1}{2}(1+\epsilon_r)}$.

Fig. 6 shows a very important property of the empirical formulas, namely that the inductances are nearly (± 1.2 percent) independant of ϵ_r, as they should be. It is interesting that this is obtained with the functions f_4–f_7 being independent of ϵ_r.

The results presented in Fig. 5(a) and (b) may be compared with the results obtained, e.g., by Bryant and Weiss [5]. The errors are nowhere greater than about 2 percent and in most cases smaller.

To explain how the empirical formulas were developed is rather difficult because the process was in fact iterative. However, an explanation is attempted below.

The results obtained by among others Bryant and Weiss [5] formed the basis for the development of the empirical formulas. The development started with the expression for w', (26), which ensures that the even-mode parameters become correct for $s = S/H \rightarrow 0$ or ∞. Then a function $f_a(w,s)$ was found so that the even-mode impedance was approximately correct when this function was substituted for $f_4(w,s)$ (28) and $f_5(w,s)$ (29) in (23) and (24). It was found that this function need not be dependant on ϵ_r. The even-mode phase velocity variation, however, was not as shown in Fig. 5 (b) which, as mentioned above, approximately shows the way the phase velocity should vary. To remedy this, corrective factors were found, changing $f_a(w,s)$ into $f_4(w,s)$ and $f_5(w,s)$, respectively. In the odd-mode case a similar development led to a function of the form giving $f_6(w,s)$ (39) (or $f_7(w,s)$ (40)) which approximately gave the odd-mode impedance. Then the functions f_c and f_d were introduced to correct the calculated phase velocity. The function $f_8(s,\epsilon_r)$ was introduced to make $\epsilon_{r2R} \rightarrow 1$ (approximately) for $s \rightarrow 0$.

IV. N-Coupled Strips

To find the two capacitance matrices $\boldsymbol{C}$ and $\boldsymbol{C}_0$ [7], [8] for N-coupled strips with and without dielectric, respectively, the N strip capacitances C_p' (defined as the ratio between charge and voltage on strip p) and their effective dielectric constants $\epsilon_{r\text{eff}p}$ (defined as the ratio between the capacitance C_p' with, and the capacitance C_{p0}' without dielectric) may be found in N different modes (voltage combinations on the strips). From the resulting $2 \cdot N^2$ equations the two matrices $\boldsymbol{C}$ and $\boldsymbol{C}_0$ can be found.

Let the N voltage combinations (modes) be represented by N column matrices (vectors) $\boldsymbol{Q}_1, \boldsymbol{Q}_2, \cdots, \boldsymbol{Q}_N$; that is, each element in a vector represents the voltage on the corresponding strip in the corresponding mode. The vectors $\boldsymbol{Q}_1, \cdots, \boldsymbol{Q}_N$ can be combined in a square matrix $\boldsymbol{Q} = \{\boldsymbol{Q}_1, \cdots, \boldsymbol{Q}_N\}$. If the N^2 capacitances found are represented by a matrix $\boldsymbol{C}'$ in which element C_{pq}' is the capacitance for strip p in mode q, then the capacitance matrices $\boldsymbol{C}$ and $\boldsymbol{C}_0$ can be found as

$$\boldsymbol{C} = \{Q_{pq} \cdot C_{pq}'\} \boldsymbol{Q}^{-1} \tag{46}$$

$$\boldsymbol{C}_0 = v_0^{-2} \{Q_{pq} \cdot v_{pq}^2 \cdot C_{pq}'\} \boldsymbol{Q}^{-1} = v_0^{-2} \cdot \boldsymbol{L}^{-1} \tag{47}$$

where $v_{pq} = v_0/\sqrt{\epsilon_{r\text{eff}pq}} = v_0 \cdot \sqrt{C_{pq0}'/C_{pq}'}$ and $\boldsymbol{L}$ is the inductance matrix for the N-coupled strips [7], [8].

The relationship between voltages on, and currents into the $2N$ ports of the network that the N-coupled strips form can be expressed as

$$\boldsymbol{I} = \begin{Bmatrix} \boldsymbol{I}_a \\ \boldsymbol{I}_b \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{Y}_{aa} & \boldsymbol{Y}_{ab} \\ \boldsymbol{Y}_{ba} & \boldsymbol{Y}_{bb} \end{Bmatrix} \begin{Bmatrix} \boldsymbol{U}_a \\ \boldsymbol{U}_b \end{Bmatrix} = \boldsymbol{YU}. \tag{48}$$

Here $\boldsymbol{U}_a$, $\boldsymbol{U}_b$, and $\boldsymbol{I}_a, \boldsymbol{I}_b$, are vectors representing the voltages on and the currents into the ports. The subindexes a and b refer to the two ends of the strips [9]. The square matrices $\boldsymbol{Y}_{aa}$, $\boldsymbol{Y}_{ab}$, $\boldsymbol{Y}_{ba}$, and $\boldsymbol{Y}_{bb}$ are admittance parameter matrices which can be expressed as

$$\boldsymbol{Y}_{aa} = \boldsymbol{Y}_{bb} \simeq \{-j \cdot Q_{pq} \cdot C_{pq}' \cdot v_{pq} \cdot \cot(\omega l/v_{pq})\} \boldsymbol{Q}^{-1} \tag{49}$$

$$\boldsymbol{Y}_{ab} = \boldsymbol{Y}_{ba} \simeq \{j \cdot Q_{pq} \cdot C_{pq}' \cdot v_{pq} \cdot \csc(\omega l/v_{pq})\} \boldsymbol{Q}^{-1} \tag{50}$$

where ω = angular frequency and l = striplength.

These expressions which are found from (46) and (47) using the results known from transmission line theory are correct for $\omega \rightarrow 0$, but not for $\omega \neq 0$. The errors, however, are small for frequencies up to about 20 percent above the quarter-wave frequency for the strips. This limit and the size of the errors are difficult to assess. For $\epsilon_r = 1$, there are no limitations in the expressions (49) and (50), but for $\epsilon_r \neq 1$ the errors become larger or the useful frequency range becomes smaller as ϵ_r increases. To give some idea of the accuracy of the method, a comparison was made between the correct y-parameters and the y-parameters of (49) and (50) for a few 3-strip circuits (the strips of equal length) and for $\epsilon_r < 10$. The capacitance matrices $\boldsymbol{C}$ and $\boldsymbol{C}_0$ were found using the empirical formulas given below. This comparison showed that the errors become more and more pronounced as the frequency gets nearer to the half-wave frequency for the strips. As a rule of thumb, the errors should be smaller than 2 percent for $\omega l/v_{pq} < 1.2\pi/2$, if the final y-parameter matrix $\boldsymbol{Y}$ (48) is taken as the mean between the calculated matrix (using (49), (50)) and its transpose. This also ensures reciprocity.

The matrix $\boldsymbol{Q}$ can be chosen almost arbitrarily, but the choice in (51) is especially attractive because each admittance parameter will consist of only two terms instead of possibly N terms, and the empirical formulas for two identical coupled strips can be modified to be used for the modes given by $\boldsymbol{Q}$.

$$\boldsymbol{Q}=\begin{Bmatrix} 1 & -1 & -1 & \cdots & -1 \\ 1 & 1 & -1 & & -1 \\ 1 & 1 & 1 & & \cdot \\ \cdot & & & & -1 \\ 1 & 1 & 1 & \cdots & 1 \end{Bmatrix}$$

$$\boldsymbol{Q}^{-1}=\begin{Bmatrix} \frac{1}{2} & 0 & 0 & \cdots & \cdot & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & 0 & & & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & & & 0 \\ \cdot & & & & & \cdot \\ 0 & 0 & 0 & \cdots & -\frac{1}{2} & \frac{1}{2} \end{Bmatrix}. \tag{51}$$

If coupling between nonadjacent strips is ignored, which in most cases is allowable, only three different modes need be considered for each strip, see Figs. 7–9.

Introducing the parameters given in Figs. 7–9, the admittance parameters (49), (50) can now be expressed as

$$y_{ppaa}=y_{ppbb}\simeq -j\tfrac{1}{2}\big(Y_{pOr}\cot(\omega l/v_{pOr}) + Y_{pOl}\cot(\omega l/v_{pOl})\big) \tag{52}$$

$$y_{ppab}=y_{ppba}\simeq j\tfrac{1}{2}\big(Y_{pOr}\csc(\omega l/v_{pOr}) + Y_{pOl}\csc(\omega l/v_{pOl})\big) \tag{53}$$

$$y_{p,p-1aa}=y_{p,p-1bb}\simeq -j\tfrac{1}{2}\big(Y_{pE}\cot(\omega l/v_{pE}) - Y_{pOl}\cot(\omega l/v_{pOl})\big) \tag{54}$$

$$y_{p,p-1ab}=y_{p,p-1ba}\simeq j\tfrac{1}{2}\big(Y_{pE}\csc(\omega l/v_{pE}) - Y_{pOl}\csc(\omega l/v_{pOl})\big) \tag{55}$$

$$y_{p,p+1aa}=y_{p,p+1bb}\simeq -j\tfrac{1}{2}\big(Y_{pE}\cot(\omega l/v_{pE}) - Y_{pOr}\cot(\omega l/v_{pOr})\big) \tag{56}$$

$$y_{p,p+1ab}=y_{p,p+1ba}\simeq j\tfrac{1}{2}\big(Y_{pE}\csc(\omega l/v_{pE}) - Y_{pOr}\csc(\omega l/v_{pOr})\big) \tag{57}$$

$$y_{p,p\pm 2}\simeq y_{p,p\pm 3}\simeq \cdots \simeq 0\,. \tag{58}$$

The mode-parameters C_{pE}, Y_{pE}, v_{pE}, etc., are found using (16)–(18) and the expressions given below for C_{1L}, C_{1R}, C_{2L}, etc. The empirical formulas for two identical coupled strips are used, but some modified strip dimensions (80)–(87) go into the formulas in order to make each gap between two strips look like the gap between two identical coupled strips.

The modified strip dimensions also ensures (approximately) that the three sets of mode parameters are calculated taking nonadjacent strips into account. However, this does not mean that coupling between nonadjacent strips have been taken fully into account because then more than three modes would be needed in the expressions (52)–(58).

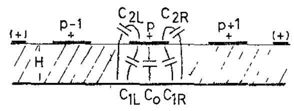


Fig. 7. Even mode. $Y_p=Y_{pE}$, $C_p=C_{pE}$, $v_p=v_{pE}$.

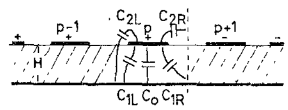


Fig. 8. Odd right mode. $Y_p=Y_{pOr}$, $C_p=C_{pOr}$, $v_p=v_{pOr}$.

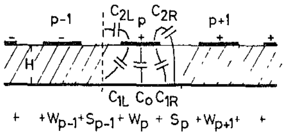


Fig. 9. Odd left mode. $Y_p=Y_{pOl}$, $C_p=C_{pOl}$, $v_p=v_{pOl}$.

Even Mode

$$C_0=C_0(w_p,\epsilon_r) \tag{59}$$
$$C_{1L}=C_1(w_{Lp},\epsilon_r)\cdot f_4(w_{Lp},s_{ELp}) \tag{60}$$
$$C_{2L}=C_2(w_{Lp},\epsilon_r)\cdot f_5(w_{Lp},s_{ELp}) \tag{61}$$
$$\epsilon_{r2L}=\epsilon_{r2}(w_{Lp},\epsilon_r) \tag{62}$$
$$C_{1R}=C_1(w_{Rp},\epsilon_r)\cdot f_4(w_{Rp},s_{ERp}) \tag{63}$$
$$C_{2R}=C_2(w_{Rp},\epsilon_r)\cdot f_5(w_{Rp},s_{ERp}) \tag{64}$$
$$\epsilon_{r2R}=\epsilon_{r2}(w_{Rp},\epsilon_r). \tag{65}$$

Odd Right Mode

$$C_0=C_0(w_p,\epsilon_r) \tag{66}$$
$$C_{1L}=C_1(w_p,\epsilon_r)\cdot f_4(w_p,s_{OrLp}) \tag{67}$$
$$C_{2L}=C_2(w_p,\epsilon_r)\cdot f_5(w_p,s_{OrLp}) \tag{68}$$
$$\epsilon_{r2L}=\epsilon_{r2}(w_p,\epsilon_r) \tag{69}$$
$$C_{1R}=C_1(w_{Rp},\epsilon_r)\cdot f_6(w_{Rp},s_{OrRp}) \tag{70}$$
$$C_{2R}=C_2(w_{Rp},\epsilon_r)\cdot f_7(w_{Rp},s_{OrRp}) \tag{71}$$
$$\epsilon_{r2R}=\epsilon_{r2}(w_{Rp},\epsilon_r)\cdot f_8(s_{OrRp},\epsilon_r). \tag{72}$$

Odd Left Mode

$$C_0=C_0(w_p,\epsilon_r) \tag{73}$$
$$C_{1l}=C_1(w_{Lp},\epsilon_r)\cdot f_6(w_{Lp},s_{OlLp}) \tag{74}$$
$$C_{2L}=C_2(w_{Lp},\epsilon_r)\cdot f_7(w_{Lp},s_{OlLp}) \tag{75}$$
$$\epsilon_{r2L}=\epsilon_{r2}(w_{Lp},\epsilon_r)\cdot f_8(s_{OlLp},\epsilon_r) \tag{76}$$
$$C_{1R}=C_1(w_p,\epsilon_r)\cdot f_4(w_p,s_{OlRp}) \tag{77}$$
$$C_{2R}=C_2(w_p,\epsilon_r)\cdot f_5(w_p,s_{OlRp}) \tag{78}$$
$$\epsilon_{e2R}=\epsilon_{r2}(w_p,\epsilon_r) \tag{79}$$

where

$$w_{Rp}=w_p+w_{p-1}/(1+s_{p-1}/w_p) \tag{80}$$
$$w_{Lp}=w_p+w_{p+1}/(1+s_p/w_p) \tag{81}$$

$$s_{ELp} = f_9(w_{Lp}, w_{Rp-1}, s_{p-1})$$
$$= s_{p-1} \cdot (1 + 2/\pi \cdot \arctan(\ln(w_{Lp}/w_{Rp-1}))) \quad (82)$$

$$s_{ERp} = f_9(w_{Rp}, w_{Lp+1}, s_p) \quad (83)$$

$$s_{OrLp} = f_9(w_p, w_{Rp-1}, s_{p-1}) \quad (84)$$

$$s_{OrRp} = f_{10}(w_{Rp}, w_{Lp+1}, s_p)$$
$$= s_p \cdot (1 + 2\pi \cdot \arctan(\ln(w_{Rp}/w_{Lp+1})/4)) \quad (85)$$

$$s_{OlLp} = f_{10}(w_{Lp}, w_{Rp-1}, s_{p-1}) \quad (86)$$

$$s_{OlRp} = f_9(w_p, w_{Lp+1}, s_p) \quad (87)$$

and

$$w_p = W_p/H \qquad s_p = S_p/H \quad \text{etc.}$$

Because empirical formulas are used, and because the expressions (52)–(58) are approximate expressions, the admittance-parameter matrix (48) will not be symmetric, as it should be, although the errors are small.

If the resulting y-parameter matrix is taken as the mean between the calculated matrix and its transpose, the errors will be even smaller, but the size of the errors is difficult to assess, because first of all it is difficult to find data on coupled microstrip circuits with more than two strips, and secondly it depends on the circuit what errors are of importance. However, as the following examples of analysis and synthesis of interdigital microstrip circuits show, the errors are so small that the method presented here of calculating the admittance-parameter matrix can be said to be a very practical and powerful tool.

A. Analysis of Microstrip Circuits

An analysis program (in ALGOL) has been developed utilizing the simplified y-parameter expressions and the empirical formulas.

In this program an equivalent circuit based on the y-parameters is made (like the π-circuit for a two-port). Into this circuit various lumped elements can be introduced (e.g., inductances to simulate nonideal short-circuits, capacitances to simulate nonideal open ends, and equivalent circuits, e.g., transistors). A standard network analysis procedure is then used on the final network.

Because lumped elements can be taken into account, the analysis program can be used on a large number of microstrip circuits: directional couplers, interdigital filters, amplifiers, etc.

In Fig. 10 a "Lange" directional coupler is shown. This coupler, the data of which are given in Table I, was analyzed using the above mentioned program. Calculated and measured responses are shown in Fig. 11. Small inductors (0.2 nH) were introduced to account for the effect of the bonding wires. There is very good agreement between measured and calculated responses. It should be mentioned that because of etching errors, the coupler did not meet the specifications.

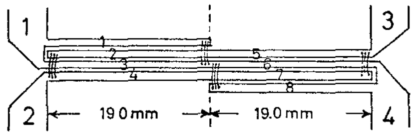


Fig. 10. Lange directional coupler.

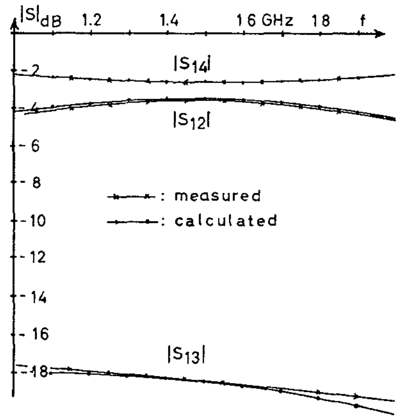


Fig. 11. Measured and calculated response for Lange directional coupler.

TABLE I
DATA FOR LANGE DIRECTIONAL COUPLER

Substrate: 0.714 mm Rogers RT/Duroid 5870, $\epsilon_r = 2.37$	
Dimensions:	(actual, not intended)
	$W_1 = W_4 = W_5 = W_8 = 0.265 \cdot H$
	$W_2 = W_3 = W_6 = W_7 = 0.308 \cdot H$
	$S_{12} = S_{34} = S_{56} = S_{78} = 0.161 \cdot H$
	$S_{23} = S_{67} = 0.165 \cdot H$

V. INTERDIGITAL BANDPASS FILTERS

The equivalent diagram for an interdigital bandpass filter (shown schematically in Fig. 12 for a 7-strip 5th-order filter) based on the admittance parameters, is shown in Fig. 13. It is assumed here that there is no coupling between nonadjacent strips ($y_{p,p\pm 2}, y_{p,p\pm 3} \cdots \simeq 0$).

If the striplengths are adjusted individually so that $y_{11aa}, y_{22bb}, \cdots, y_{NNaa}$ are all 0 at the center frequency f_0 for the filter, then from (52)–(58) and after making sure of reciprocity (letting $Y = (Y + Y^T)/2$):

at frequencies near f_0:

$$y_{ppaa} \simeq -\tfrac{1}{2}(Y_{pOl} + Y_{pOr}) \frac{\pi BW}{4\omega_0} s' = K_p s' \quad (88)$$

$$y_{p,p+1ab} \simeq (Y_{pE} - Y_{pOr} + Y_{p+1E} - Y_{p+1Ol})/4 = jJ_{p,p+1} \quad (89)$$

where

$$s' = j \cdot 2(\omega - \omega_0)/BW \quad (90)$$

$$BW = \text{bandwidth in rad/s.}$$

The equivalent diagram in Fig. 13 now represents N resonators ($y_{pp..}$) with admittance converters in between. Normally, y_{11aa} and $y_{NN..}$ will be much smaller than $1/Z_0$, which means that an N-strip interdigital filter of the type shown in Fig. 12 is an $N-2$ order filter.

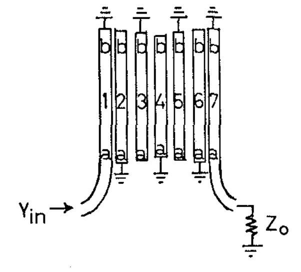


Fig. 12. N-strip $N-2$ order interdigital bandpass filter, shown here schematically for $N=7$.

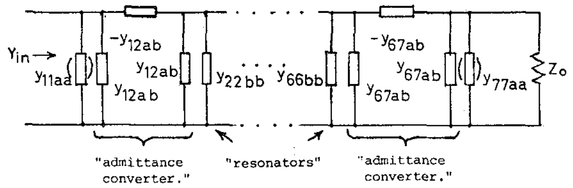


Fig. 13. Equivalent diagram for interdigital bandpass filter of the type shown in Fig. 12, $N=7$. Coupling between nonadjacent strips is ignored.

Synthesizing the resonator constants K_p and the admittance converter constants $J_{p,p+1}$ to give the desired filter response (e.g., a Chebyshev response) is classical and described, e.g., in [10]. The combination of stripwidths and spacings, however, that give these K's and J's, must be found by iteration. Fortunately this is no serious problem because the K's depend mainly on the corresponding stripwidths, and the J's depend mainly on the corresponding strip-spacings.

A synthesis program (in ALGOL) has been developed, which can synthesize interdigital bandpass filters of different types, one of which is shown schematically in Fig. 12. The input data are: equivalent lowpass filter constants, bandwidth, an initial set of strip-dimensions, and of course, the substrate thickness and dielectric constant. The output is a combination of dimensions that will give the desired filter response. This combination, however, need not be practical (there are an infinite number of possible solutions). It may, therefore, be necessary to do a few runs before a practical combination is found.

The computing time is on the order of 2 s for a 4th-order filter (on an IBM 370-165).

A. 5th-Order Chebyshev Filter

For the filter shown in Fig. 14, the desired specifications were: center frequency = 670 MHz, 20-percent bandwidth, and 5th-order Chebyshev response with 1-dB ripple. This filter was made to test the synthesis procedure and the empirical formulas. Measured and calculated response is shown in Fig. 15 and show good agreement. Open-end effects and nonideal shortcircuits were accounted for by making the striplengths 3.7-mm shorter than the calculated lengths. The physical data for the filter are given in Table II. The calculated response was obtained from these data, using the above-mentioned analysis program. Capacitors (0.14 pF) were introduced to simulate open-end effects and inductors (0.5 nH) to simulate nonideal short circuits. Losses have been taken into account by assuming the same attenuation constant ($\alpha = 0.025$ N/m) for all propagation modes.

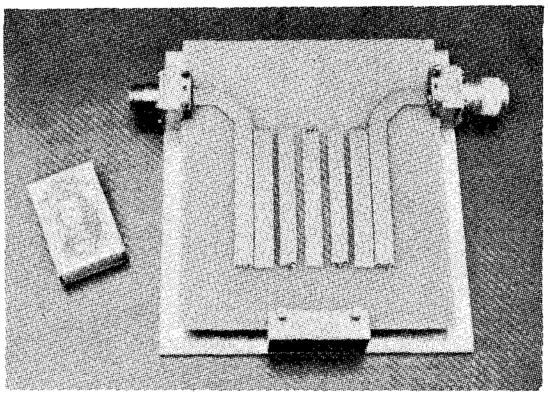


Fig. 14. Seven strip 5th-order Chebyshev bandpass filter.

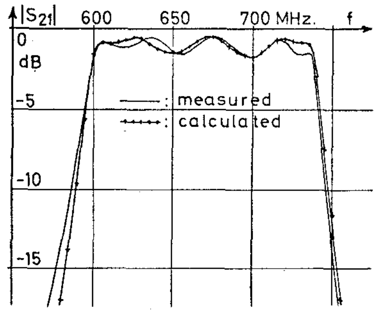


Fig. 15. Response for 5th-order Chebyshev filter.

TABLE II
DATA FOR 5TH-ORDER CHEBYSHEV FILTER

Substrate: 1/8" REXOLENE S, $\epsilon_r = 2.32$.
Dimensions:

p	1	2	3	4	5	6	7
w_p	3.00	3.10	2.65	2.83	2.65	3.10	3.00
s_p	0.22	1.08	1.44	1.44	1.08	0.22	∞
l_p	76.6	76.6	76.0	75.7	76.0	76.6	76.6 mm.

B. 4th-Order Chebyshev Filter

The filter shown in Fig. 16 is used in a QPSK microwave-communication system made at the Electromagnetic Institute at the Technical University of Denmark, Copenhagen. The desired specifications for this filter were: center frequency = 1.5 GHz, 100-MHz bandwidth, and 4th-order Chebyshev response with 0.1-dB ripple. The resulting physical data for the filter are given in Table III. The striplengths were made 0.4-mm shorter than the calculated lengths to account for open-end capacitances. It was assumed that the short-circuit inductances were small, but as it can be seen from Fig. 17 in which the filter response is shown, the center frequency lies 1 percent below 1.5 GHz. In the analysis of the filter, it was

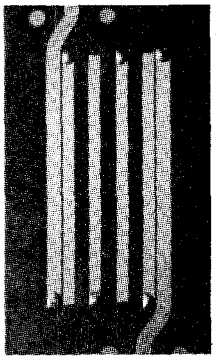


Fig. 16. Six strip 4th-order Chebyshev bandpass filter.

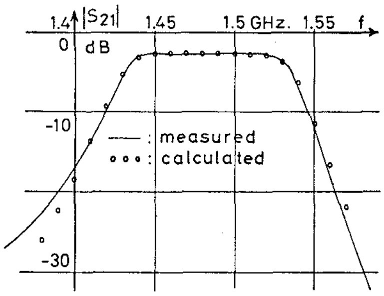


Fig. 17. Response for 4th-order Chebyshev bandpass filter.

TABLE III
DATA FOR 4TH-ORDER CHEBYSHEV FILTER

Substrate: 0.714 mm. ROGERS RT/DUROID 5870, $\epsilon_r = 2.37$.
Dimensions:

p	1	2	3	4	5	6
w_p	2.80	2.27	2.27	2.27	2.27	2.80
s_p	0.455	3.00	3.30	3.00	0.455	∞
l_p	36.2	35.9	35.4	35.4	35.9	36.2 mm.

assumed that the short-circuit inductances were 0.086 nH. It was also assumed that the attenuation constant for all propagation modes was 0.15 N/m. The agreement between measured and calculated response is quite good. This is noteworthy, because the small relative bandwidth of 6.7 percent leads to large strip spacings, about 3 times the substrate thickness, and data on coupling between strips that far from each other are not very reliable.

VI. Conclusion

The empirical formulas presented give quite accurate propagation-mode parameters for single microstrips, two identical coupled strips, and for multiple-coupled strips. The formulas form the basis for approximate admittance parameter expressions for the $2N$-port network that the N-coupled strips form. These expressions again form the basis for accurate analysis and synthesis programs for microstrip circuits.

Acknowledgment

The author wishes to thank F. Hansen and others at the Electromagnetic Institute at the Technical University of Denmark, Copenhagen, for their invaluable help. He also wishes to thank T. Larsen for his critical reading of the manuscript.

References

[1] H. A. Wheeler, "Transmission-line properties of a strip on a dielectric sheet on a plane," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 631–647, Aug. 1977.

[2] S. V. Judd *et al.*, "An analytical method for calculating microstrip transmission line parameters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 78–87, Feb. 1970.

[3] M. Ramadan and Westgate, "Impedance of coupled microstrip transmission lines," *Microwave J.*, pp. 30–34, July 1971.

[4] Sina Akthtarzad *et al.*, "The design of coupled microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 486–492, 1975.

[5] G. B. Bryant and J. A. Weiss, "Parameters of microstrip transmission lines and of coupled pairs of microstrip lines." *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 1021–1027, Dec. 1968.

[6] W. T. Weeks, "Calculation of coefficients of capacitance of multiconductor transmission lines in the presence of a dielectric interface," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 35–43, Jan. 1970.

[7] Roland Briechle, "Analyse von Kammleitungsfiltern und ähnlichen Baugruppen aus gekoppelten Mehrleitersystemen," *FREQUENZ* 29, no. 4, pp. 94–100, Apr. 1975.

[8] Roland Briechle, "Kammleitungsfilter aus gekoppelten Mikrostreifenleitungen," *FREQUENZ* 30, no. 8, pp. 209–219, Aug. 1976.

[9] A. B. Dalby, "Analysis and synthesis of microstrip circuits with more than two coupled strips," *7th European Microwave Conf., Book of proc.*, pp. 441–445, Sept. 1977. A revised copy of this paper can be obtained from the author.

[10] R. E. Collin, *Foundations for Microwave Engineering.* New York: McGraw-Hill, 1965.